Storage-ring free-electron-laser dynamics and head-tail instability

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We develop a heuristic dynamical model to analyze the interplay between free-electron-laser (FEL) storage-ring dynamics and instabilities of the head-tail type. We show that, under given conditions, the FEL may inhibit the onset of the instability and may provide a reduction of the electron-beam transverse dimensions. Some experimental results that can be used in support of the model are also reported.
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I. INTRODUCTION

One of the intriguing features of the storage-ring-based free-electron-laser (FEL) dynamics is the interplay between the laser mechanism and the beam instabilities [1]. Previous experimental investigations have reported evidence according to which the FEL may provide a feedback mechanism that inhibits the onset of some instabilities [2]. A successive theoretical analysis [3] has shown that the microwave instability or anomalous bunch lengthening [4] may be counteracted by the FEL interaction. The mechanisms invoked for the physical explanation of the effect are complex, but, roughly speaking, they can be traced back to the FEL-induced energy spread, which causes a shift of the threshold of the instability, and to a more subtle effect associated with a faster damping of the higher-order longitudinal distribution modes, due to a variation of the damping times determined by the interaction itself. Experimental observations [1] have also pointed out that the head-tail instability may be affected by the FEL interaction and eventually be damped if some operating conditions are satisfied.

This paper is aimed at clarifying how and under what conditions a FEL-type interaction may act as a stabilizing feedback for the head tail-instability. The analysis we will develop is based on the heal-tail instability treatment described in Ref. [4] and on the rate equation storage-ring FEL model of Refs. [5]. The paper is organized as follows. In Sec. II we describe the model and its main consequences. Section III contains concluding remarks and some experimental results that could be interpreted using the arguments developed in the paper.

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II. HEAD-TAIL INSTABILITY AND FEL DYNAMICS

The head-tail instability is characterized by a rise time depending on the characteristic parameters of the machine and in particular on the phase [4]

$$\chi=\frac{2\pi\xi v_\beta\sigma_z}{\alpha_c c}, \tag{1}$$

where α_c is the momentum compaction, ξ the chromaticity factor, v_β the betatron frequency, and σ_z the longitudinal bunch length. The head-tail instability may induce a growth of the displacement of the transverse beam position or a growth of the transverse beam dimensions.

According to Ref. [4], the growth rates of the beam displacement position and of the transverse dimensions, under the assumption of a water bag approximation with a broadband impedance, are respectively given by

$$\frac{1}{\tau_p}=-\frac{1}{\tau_0}\int_{-\infty}^{+\infty}d\omega'\,\mathrm{Re}[\overline{z^{\perp}}(\omega')]J_0^2\left(\omega'\frac{\sigma_z}{c}-\chi\right),$$

$$\frac{1}{\tau_d}=-\frac{1}{\tau_0}\int_{-\infty}^{+\infty}d\omega'\,\mathrm{Re}[\overline{z^{\perp}}(\omega')]J_1^2\left(\omega'\frac{\sigma_z}{c}-\chi\right), \tag{2}$$

$$\tau_0=\frac{8\pi^2}{\mathcal{N}_e}\,\frac{\gamma T v_\beta b^2}{w_0 r_0 c},$$

where τ_0 is a characteristic time, $\mathcal{N}_e$ is the number of electrons in the beam, T is the machine revolution period, b is the vacuum pipe radius, w_0 is the wake-field amplitude, r_0 is the electron classical radius, $J_{0,1}(\ )$ are ordinary cylindrical

TABLE I. Notation.

I	intracavity intensity
$I_s(\mathrm{MW/cm^2})=6.9\times 10^2\left(\frac{\gamma}{N}\right)^4 \frac{1}{[\lambda_u(\mathrm{cm})kf_b(\xi)]^2}$	FEL saturation intensity
k	undulator parameter
$f_b(\xi)$	Bessel factor gain correction
$y=\mu\ (I/I_s)$	dimensionless intracavity intensity
N	number of undulator periods
$\mu=\left(\frac{0.433}{N}\right)^2 \frac{\beta}{4}\frac{\tau_s}{T}\frac{1}{\sigma_\epsilon^2(0)}, \quad \beta=\frac{\pi}{2}1.0145$	
T	revolution machine period
τ_s	longitudinal damping time
$\sigma_\epsilon(0)$	natural energy spread
N	number of undulator periods
$\mu_\epsilon(0)=4N\sigma_\epsilon(0)$	
$\tilde{\sigma}$	ratio between induced and natural energy spread
g_0	small signal gain coefficient
$T_0=\frac{T}{0.85g_0}$	
η	cavity losses
$r=\eta/0.85g_0$	

Bessel functions, and $\overline{z^\perp}(\omega)$ denotes the ω-dependent part of the wake-field impedence, which we will assume to be of the resistive-inductive-capacitive form

$$\overline{z^\perp}(\omega)=\frac{1}{\omega\left[1+i\left(\frac{\omega}{\omega_R}-\frac{\omega_R}{\omega}\right)\right]}, \tag{3}$$

with ω_R the resonant frequency of the broadband impedence. The head-tail instability model we are considering is not characterized by a threshold and instability develops only if the growth rates are positive. We have not included mechanisms leading to the saturation of the instability itself; we consider for the moment its linear part only and assume that the FEL and head-tail dynamics are switched on at the same time. Further comments on these assumptions are presented in Sec. III.

It is evident that if the head-tail instability is active the gain of a FEL is reduced by the consequent variation of the filling factor. The simplest assumption we can do is that the gain varies with the time according to

$$G\propto \exp\left(-\frac{t}{\tau_{p,d}}\right). \tag{4}$$

We can take into account the following FEL head-tail feedback loop: (a) the FEL interaction induces a bunch lengthening, (b) the induced bunch lengthening modifies the head-tail phase [see Eq. (1)], (c) this phase variation provides a modification of the growth rates through Eqs. (2), and (d) the growth rate modification yields a FEL gain variation according to Eq. (4).

To derive the evolution of a FEL oscillator with the inclusion of the head-tail instability we marginally modify the rate equation model of Ref. [5]. Indeed, we use the coupled equations

$$\frac{dy}{d\tau}=\frac{y\tau_s}{T_0}\left\{\frac{1}{\sqrt{1+\tilde{\sigma}^2}}\frac{e^{-\tau/\tau_{p,d}(\tilde{\sigma}^2)}}{[1+1.7\mu_\epsilon^2(0)(1+\tilde{\sigma}^2)]}-r\right\}, \tag{5a}$$

$$\frac{d\tilde{\sigma}^2}{d\tau}=-2(\tilde{\sigma}^2-y), \tag{5b}$$

where τ is a dimensionless time normalized to the damping time τ_s, y is linked to the FEL optical power, and $\tilde{\sigma}$ is the ratio between the induced and the natural energy spread. The exponential in Eq. (5a) takes into account the gain variation due to the head-tail instability and the growth rate depends on the induced energy spread through the phase χ_0, which should be redefined as

$$\chi=\chi_0\sqrt{1+\tilde{\sigma}^2}, \tag{6}$$

TABLE II. Numerical values.

$g_0=0.1$
$N=40$
$\tau_s=1.5$ ms
$\tau_0=1$ ms
$T=240$ ns
$r=0.2$

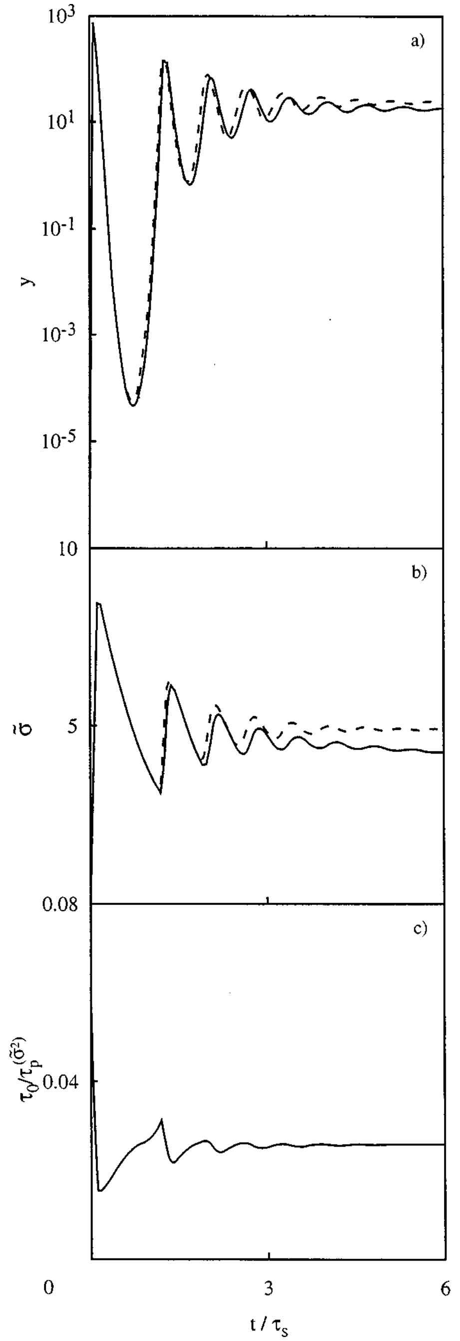

FIG. 1. (a) Dimensionless intracavity power vs t/τ_s without (dotted line) and with (continuous line) head-tail instability for $\chi_0 = -0.05$, $\tau_0 = 1$ ms and the parameters of Table II. (b) Induced energy spread normalized to the natural energy spread for the same parameters in (a). (c) $\tau_0/\tau_p(\tilde{\sigma}^2)$ vs t/τ_s.

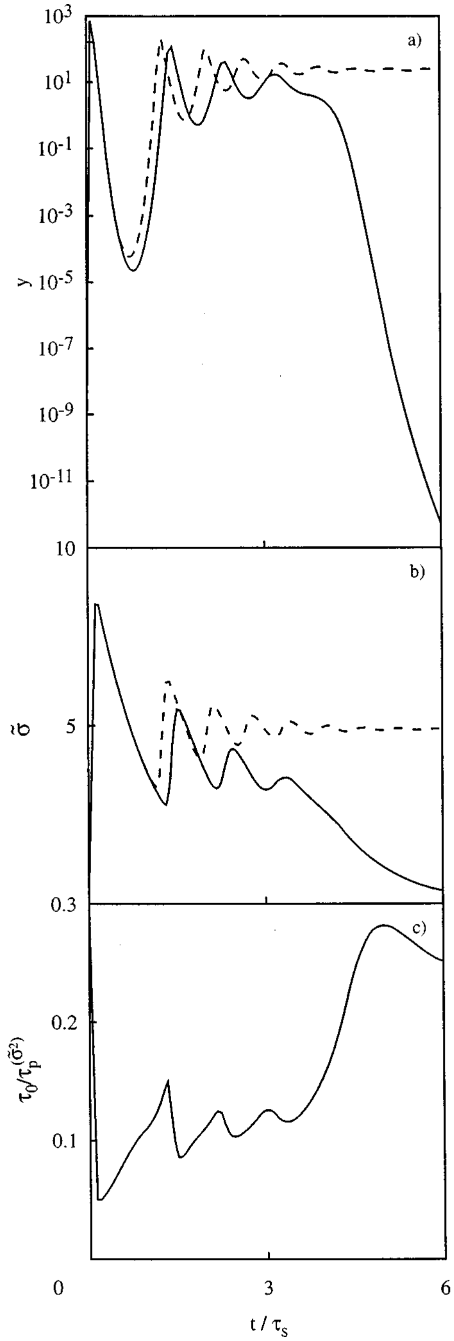

FIG. 2. Same as Fig. 1 but for $\chi_0 = -0.2$.

with χ_0 being the head-tail phase, i.e., that in the absence of a FEL interaction. For the other parameters appearing in Eqs. (5) see Table I.

The first example we consider is the case in which the instability is dominated by the displacement term. This mode is unstable for negative value of χ_0. We use $\chi_0 = -0.05$ and -0.2, in addition to the other values in Table II, to solve Eqs. (5).

In Fig. 1 we report the evolution of the dimensionless intracavity power [Fig. 1(a)], the induced energy spread [Fig. 1(b)], and the growth rate $1/\tau_p$ normalized to $1/\tau_0$ [Fig. 1(c)]. Figures 1(a) and 1(b) contain a comparison between the cases $\chi_0 = -0.05$ and 0 (i.e., without head-tail instability). It is evident that in this case the FEL may survive for a rather long time. Indeed, the interaction moves the growth rate to a small (positive) value around zero; however, as time increases the laser decays and eventually is switched off. In Fig. 2 we have considered the case $\chi_0 = -0.2$ and it is evident that for this value, the FEL is not able to counteract the head-tail instability and the laser is switched off in a rather short time.

In Fig. 3 we show the analog of Figs. 1 and 2 for the instability dominated by the transverse-dimension growth. It is evident that in this case the FEL reaches a completely

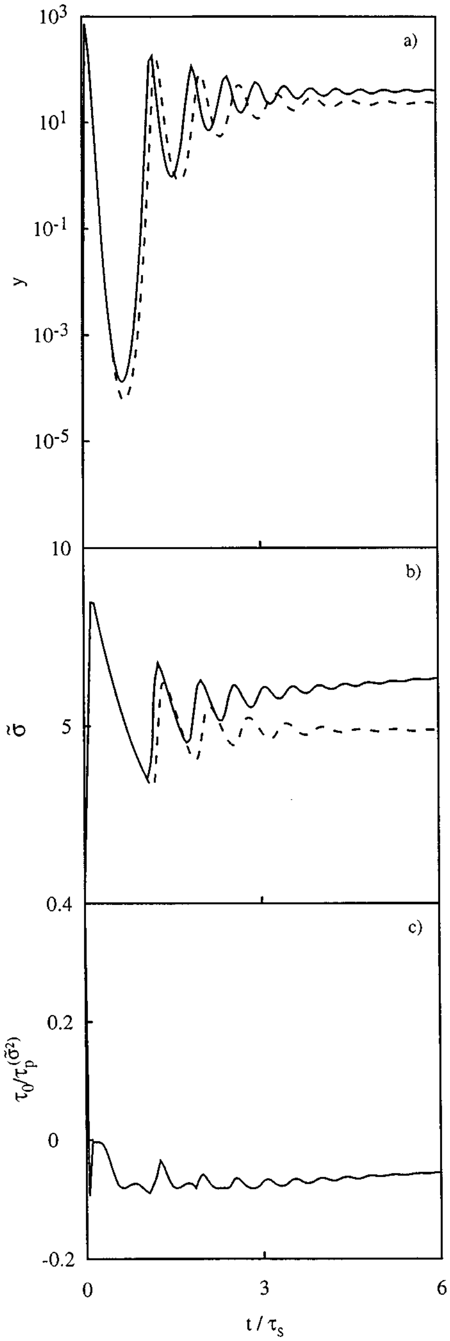

FIG. 3. Same as Fig. 1 but for $\chi_0=1$.

stable behavior and is able to fully counteract the instability. Indeed Fig. 3(c) shows that the eigenvalue becomes negative, thus providing a fast damping.

III. CONCLUDING REMARKS

Possible experimental evidence of the inhibition of the transverse-dimension excitation has been observed at Orsay with the super-ACO FEL where the FEL has stabilized the transverse excitation of the positron beam. The head-tail nature of the instability observed at Orsay has been clarified by a proper analysis of its dependence on the machine chromaticity. When the instability is active the beam transversally blows up and is particularly noisy. The onset of the FEL provides a reduction of the noise and of the transverse dimensions. Figures 4 and 5 yield an idea of the effect of the laser on the beam. When the laser is on, the horizontal rms width is reduced and the vertical shape is made more regular (albeit with longer tails) (see Fig. 4). The stabilization of the transverse dimensions is observed when the longitudinal packet length increases, as shown in Fig. 5, where the laser-induced lengthening is reported along with the optical pulses.

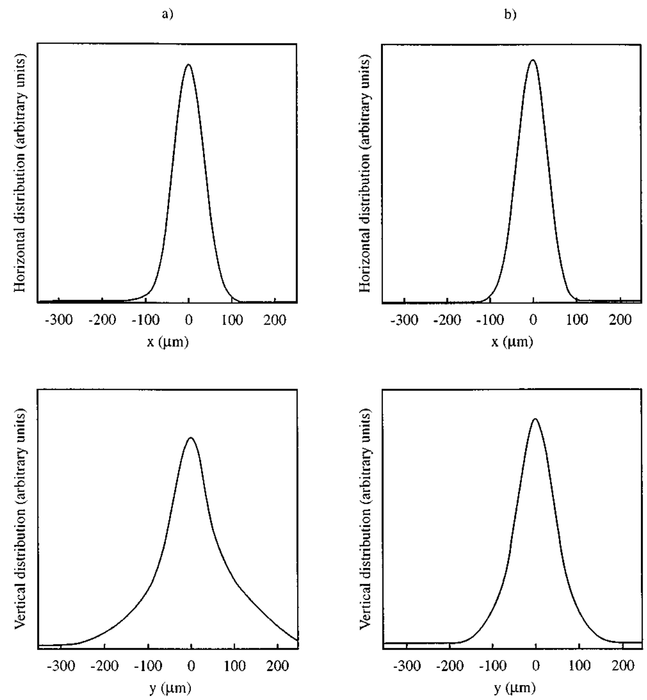

FIG. 4. Electron-beam transverse distribution for (a) 31.4 mA with the laser off, $\sigma_x=202$ μm, and $\sigma_y=342$ μm and 31.5 mA, with the laser on, $\sigma_x=197$ μm, and $\sigma_y=288$ μm.

It should be understood that the present discussion is just an indication that the FEL and the head-tail instability may have some kind of mutual feedback. The assumption we have made, however, is rather crude and deserves further comment. In particular we have tacitly assumed that by fixing the sign of the natural head-tail phase, one mode is stable and the other is unstable. This is true for small χ values only and, according to the model we have used in this paper, the p mode is stable (unstable) according to whether $\chi_0>0$ ($\chi_0<0$). The d mode may be stable or unstable, depending on the value of χ. For positive χ values the motion is stable for $2<\chi<4$ and unstable for $0<\chi<2$ and vice versa for negative χ (see Fig. 6).

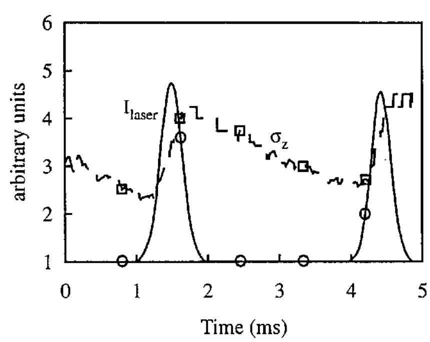

FIG. 5. Optical pulses and induced electron-bunch lengthening.

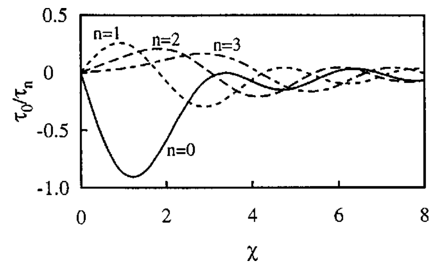

FIG. 6. τ_0/τ_n vs χ. $n=0$ and 1 should be associated with p and d modes, respectively, and $n=2,3,...$ represents higher-order modes. The growth rates of the higher-order modes are obtained from Eqs. (2) by replacing $J_{0,1}(\ )$ with $J_n(\ )$.

It is evident that even starting with a χ_0 value, yielding a stable d motion, the FEL interaction may modify the phase χ and move the system into the unstable phase region. Figures 1 and 2 should be reconsidered by taking this fact into account too and they are merely indicative. Figure 3 is more realistic and we must underline that the experimental situation is explained by taking into account an instability of d type.

We have also considered a case in which both modes are initially unstable. We must underline that in this hypothesis even choosing $\chi_0=-1.1$ and large values of τ_0 (4 ms) the instability is always dominant and the FEL is switched off.

In the previous analysis we have assumed a fairly large value of the small-signal gain coefficient ($g_0\sim 0.1$). This assumption leads to a large induced energy spread and a large value of the associated bunch lengthening. We want to emphasize that the suppression of the head-tail instability may occur even for smaller gain values. We have considered $g_0 \cong 3.7\%$ for case d and have found a suppression of the instability even though the induced energy spread is significantly smaller than the previous cases.

As a further comment let us note that the present analysis includes p and d (dipolar and quadrupolar) modes only. We did not include the effect of higher-order modes (see Fig. 4). We must underline that their effect on the gain is secondary unless the transverse distribution is distorted in a significant way.

The inclusion of these terms requires a more complete three-dimensional treatment, demanding the combined evolution of the transverse electron-beam distribution and transverse laser modes. This analysis requires considerable computational effort and is under consideration.

Let us finally underline that our analysis does not include any effect of saturation of the instability itself. In the case of the head-tail effect the instability may grow without feedback and then the beam is lost (or becomes useless). On the other hand, the system may reach a kind of a pseudostationary noisy state oscillating around an equilibrium position. The oscillation amplitude depends on the electron-beam length and the onset of the FEL may reduce the amplitude of the transverse oscillations.

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